Floating-point control in the Intel® C/C++ compiler and libraries
or
Why doesn’t my application always give the same answer?

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Agenda

- Overview
- Floating Point (FP) Model
  - Comparisons with gcc
- Performance impact
- Runtime math libraries
- Intel® Xeon Phi™ Coprocessors – what’s different
Overview

• The finite precision of floating-point operations leads to an inherent uncertainty in the results of a floating-point computation
  – Results may vary within this uncertainty

• Nevertheless, may need reproducibility beyond this uncertainty
  – For reasons of Quality Assurance, e.g. when porting, optimizing, etc

• The right compiler options can deliver consistent, closely reproducible results whilst preserving good performance
  – Across IA-32, Intel® 64 and other IEEE-compliant platforms
  – Across optimization levels
  – -fp-model is the recommended high level control for the Intel Compiler
Floating Point (FP) Programming Objectives

- **Accuracy**
  - Produce results that are “close” to the correct value
    - Measured in relative error, possibly in ulp

- **Reproducibility**
  - Produce consistent results
    - From one run to the next
    - From one set of build options to another
    - From one compiler to another
    - From one platform to another

- **Performance**
  - Produce the most efficient code possible

These options usually conflict!
Judicious use of compiler options lets you control the tradeoffs.
Different compilers have different defaults.
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Floating Point Semantics

• The –fp-model (/fp:) switch lets you choose the floating point semantics at a coarse granularity. It lets you specify the compiler rules for:
  
  – **Value safety** (main focus)
  – FP expression evaluation
  – FPU environment access
  – Precise FP exceptions
  – FP contractions (fused multiply-add)

• Also pragmas in C99 standard
  – #pragma STDC FENV_ACCESS etc

• Old switches such as –mp now deprecated
  – Less consistent and incomplete; don’t use
The `-fp-model` switch for icc

• `-fp-model`
  - `fast [=1]` allows value-unsafe optimizations (default)
  - `fast=2` allows additional approximations
  - `precise` value-safe optimizations only
  - `source | double | extended` imply “precise” unless overridden
    see “FP Expression Evaluation” for more detail
  - `except` enable floating point exception semantics
  - `strict` precise + except + disable fma +
    don’t assume default floating-point environment

• Replaces old switches `-mp`, `-fp-port`, etc (don’t use!)

• `-fp-model precise -fp-model source`
  – recommended for ANSI/IEEE standards compliance, C+
    + & Fortran
  – “source” is default with “precise” on Intel 64 Linux
GCC option

- \texttt{-f[no-]fast-math} is high level option
  - It is \textit{off by default} (different from icc)
  - It is turned on by \texttt{-Ofast}

- Components control similar features:
  - Value safety (-funsafe-math-optimizations)
    - includes reassociation
  - Reproducibility of exceptions
  - Assumptions about floating-point environment
  - Assumptions about exceptional values

- also sets abrupt/gradual underflow (FTZ)

- For more detail, check backup or \url{http://gcc.gnu.org/wiki/FloatingPointMath}
Value Safety

- In SAFE mode, the compiler may not make any transformations that could affect the result, e.g. all the following are prohibited.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x / x)</td>
<td>(1.0)</td>
<td>(x) could be 0.0, (\infty), or NaN</td>
</tr>
<tr>
<td>(x - y)</td>
<td>(-(y - x))</td>
<td>If (x) equals (y), (x - y) is +0.0 while (- (y - x)) is -0.0</td>
</tr>
<tr>
<td>(x - x)</td>
<td>0.0</td>
<td>(x) could be (\infty) or NaN</td>
</tr>
<tr>
<td>(x \times 0.0)</td>
<td>0.0</td>
<td>(x) could be -0.0, (\infty), or NaN</td>
</tr>
<tr>
<td>(x + 0.0)</td>
<td>(x)</td>
<td>(x) could be -0.0</td>
</tr>
<tr>
<td>((x + y) + z)</td>
<td>(x + (y + z))</td>
<td>General reassociation is not value safe</td>
</tr>
<tr>
<td>((x == x))</td>
<td>true</td>
<td>(x) could be NaN</td>
</tr>
</tbody>
</table>
Value Safety

Affected Optimizations include:

• Reassociation
• Flush-to-zero
• Expression Evaluation, various mathematical simplifications
• Approximate divide and sqrt
• Math library approximations
Reassociation

- Addition & multiplication are “associative” (& distributive)
  - a+b+c = (a+b) + c = a + (b+c)
  - a*b + a*c = a * (b+c)

- These transformations are equivalent **mathematically**
  - but **not** in finite precision arithmetic

- Reassociation can be disabled in its entirety
  - ⇒ for standards conformance ( C left-to-right )
  - Use **-fp-model precise**
    - May carry a significant performance penalty
      (other optimizations also disabled)

- Parentheses are respected only in value-safe mode!
  - -assume protect_paren composite (Fortran only)

- See exercises for an example derived from a real app
Example  (see exercises)

“tiny” is intended to keep $a[i]>0$

but... optimizer hoists constant expression $(c+\text{tiny})$ out of loop

$\text{tiny}$ gets “rounded away” wrt $c$

```
icc -O1 reassoc.cpp; ./a.out
a = 0  b = inf
icc -fp-model precise reassoc.cpp; ./a.out
a = 1e-20  b = 1e+20
```

g++ reassoc.cpp; ./a.out
a = 1e-20  b = 1e+20
g++ -O3 -ffast-math reassoc.cpp; ./a.out
a = 0  b = inf

```
#include <iostream>
#define N 100

int main()  {
    float a[N], b[N];
    float c = -1., tiny = 1.e-20F;

    for (int i=0; i<N; i++) a[i]=1.0;
    for (int i=0; i<N; i++)  {
        a[i] = a[i] + c + tiny;
        b[i] = 1/a[i];
    }
    std::cout << "a = " << a[0] << "   b = " << b[0] << "\n";
}
```
Denormalized numbers and Flush-to-Zero (FTZ)

- Denormals extend the (lower) range of IEEE floating-point values, at the cost of:
  - Reduced precision
  - Reduced performance (can be 100 X for ops with denormals)

- If your application creates but does not depend on denormal values, setting these to zero may improve performance ("abrupt underflow", or "flush-to-zero",)
  - Done in Intel® SSE or Intel® AVX hardware, so fast
  - Happens by default at –O1 or higher (for icc, not gcc)
  - -no-ftz or –fp-model precise will prevent
    - Must compile main with this switch to have an effect
    - -fp-model precise –ftz to get “precise” without denormals
  - Not available for x87, denormals always generated
    - (unless trapped and set to zero in software – very slow)

- For gcc, -ffast-math sets abrupt underflow (FTZ)
  - But –O3 -ffast-math reverts to gradual underflow
Reductions

• Parallel implementations imply reassociation (partial sums)
  – Not value safe, but can give substantial performance advantage
  – -fp-model precise
    – disables vectorization of reductions
      – except those mandated by Intel® Cilk Plus
    – does not affect OpenMP* or MPI* reductions
      These remain value-unsafe
      (programmer’s responsibility)

```c
float Sum(const float A[], int n)
{
  float sum=0;
  for (int i=0; i<n; i++)
    sum = sum + A[i];
  return sum;
}
```

```c
float Sum( const float A[], int n )
{
  int i, n4 = n-n%4;
  float sum=0,sum1=0,sum2=0,sum3=0;
  for (i=0; i<n4; i+=4) {
    sum  = sum  + A[i];
    sum1 = sum1 + A[i+1];
    sum2 = sum2 + A[i+2];
    sum3 = sum3 + A[i+3];
  }
  sum = sum + sum1 + sum2 + sum3;
  for (; i<n; i++) sum = sum + A[i];
  return sum;
}
```
Reproducibility of Reductions in OpenMP*

• Each thread has its own partial sum
  – Breakdown, & hence results, depend on number of threads
  – Partial sums are summed at end of loop
  – Order of partial sums is undefined (OpenMP standard)
    – First come, first served
    – Result may vary from run to run (even for same # of threads)
    – For both gcc and icc
    – Can be more accurate than serial sum
  – For icc & ifort, option to define the order of partial sums (tree algorithm)
    – Makes results reproducible from run to run
    – export KMP_DETERMINISTIC_REDUCTION=yes (in 13.0)
      – May also help accuracy
      – Possible slight performance impact, depends on context
    – Requires static scheduling, fixed number of threads
    – Default for large numbers of threads
FP Expression Evaluation

In the following expression, what if a, b, c, and d are mixed data types (single and double for example)

\[ a = (b + c) + d \]

Four possibilities for intermediate rounding, (corresponding to C99 FLT_EVAL_METHOD)

- Indeterminate (-fp-model fast)
- Use precision specified in source (-fp-model source)
- Use double precision (C/C++ only) (-fp-model double)
- Use long double precision (C/C++ only) (-fp-model extended)

- Or platform-dependent default (-fp-model precise)
  - Defaults to -fp-model source on Intel64
  - Recommended for most purposes
- The expression evaluation method can significantly impact performance, accuracy, and portability
The Floating Point Unit (FPU) Environment

- FP Control Word Settings
  - Rounding mode (nearest, toward $+\infty$, toward $-\infty$, toward 0)
  - Exception masks, status flags
    (inexact, underflow, overflow, divide by zero, denormal, invalid)
  - Flush-to-zero (FTZ), Denormals-are-zero (DAZ)
  - x87 precision control (single, double, extended) [don’t mess!]

- Affected Optimizations, e.g.
  - Constant folding (evaluation at compile time)
  - FP speculation
  - Partial redundancy elimination
  - Common subexpression elimination
  - Dead code elimination
  - Conditional transform, e.g.
    
    if (c) x = y; else x = z; $\Rightarrow$ x = (c) ? y : z;
FPU Environment Access

- When access disabled (default):
  - compiler assumes default FPU environment
    - Round-to-nearest
    - All exceptions masked
    - No FTZ/DAZ
  - Compiler assumes program will NOT read status flags

- If user might change the default FPU environment, inform compiler by setting FPU environment access mode!!
  - Access may only be enabled in value-safe modes, by:
    - `-fp-model strict`
    - `#pragma STDC FENV_ACCESS ON`
  - Compiler treats control settings as unknown
  - Compiler preserves status flags
  - Some optimizations are disabled
- If you forget this, you might get **completely** wrong results!
  - Eg from math functions, if you change default rounding mode
Example

double x, zero = 0.;

feenableexcept
(FE_DIVBYZERO);

for( int i = 0; i < 20; i++ )
    x = zero ? (1./zero) :
        zero;

Problem: F-P exception from (1./zero) despite explicit protection
- The invariant (1./zero) gets speculatively hoisted out of loop by optimizer, but the "?" alternative does not
- Compiler thinks safe, because exceptions are masked by default
- Exception occurs before the protection can kick in
  - May not occur for Intel® AVX, which have masked vector instructions

Solution: Disable optimizations that lead to the premature exception
- icc -fp-model strict
  Warns compiler that F-P defaults have been modified
- #pragma STDC FENV_ACCESS ON does likewise
- icc -fp-speculation safe
  Disables just speculation where this could cause an exception
Precise FP Exceptions

- When Disabled (default):
  - Code may be reordered by optimization
  - FP exceptions might not occur in the “right” places

- When enabled by
  -fp-model strict
  -fp-model except
  #pragma float_control(except, on)
  - The compiler must account for the possibility that any FP operation might throw an exception
    - Disables optimizations such as FP speculation
    - May only be enabled in value-safe modes
    - (more complicated for x87)
  - Does not unmask exceptions
    - Must do that separately, e.g.
      -fp-trap=common for C
      or functions calls such as feenableexcept()
      -fpe0 or ieee_set_haltin_q_mode() for Fortran
Floating Point Contractions

- affects the generation of FMA instructions on Intel® MIC architecture and Intel® AVX2 (-xcore-avx2)
  - Enabled by default or -fma, disable with –no-fma
  - Disabled by –fp-model strict or C/C++ #pragma
  - NOT disabled by –fp-model precise
  - -[no-]fma switch overrides –fp-model setting
  - Intel compiler does NOT support 4-operand AMD*-specific fma instruction)

- When enabled:
  - The compiler may generate FMA for combined multiply/add
    - Faster, more accurate calculations
    - Results may differ in last bit from separate multiply/add

- When disabled:
  - -fp-model strict, #pragma fp_contract(off) or –no-fma
  - The compiler must generate separate multiply/add with intermediate rounding
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Typical Performance Impact of -fp-model source

- Measured on SPECCPU2006fp benchmark suite:
- -O2 or -O3
- Geomean reduction due to
  -fp-model precise -fp-model source
  in range 12% - 15%

- Intel® Compiler XE 2011 (12.0)
- Measured on Intel Xeon® 5650 system with dual, 6-core processors at 2.67Ghz, 24GB memory, 12MB cache, SLES* 10 x64 SP2

Use -fp-model source (/fp:source) to improve floating point reproducibility whilst limiting performance impact
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Math Library Functions

- Different implementations may not have the same accuracy
  - On Intel 64:
    - libsvml for vectorized loops
    - libimf (libm) elsewhere
    - Processor-dependent code within libraries, selected at runtime
    - Inlining was important for Itanium, to get software pipelining, but less important for Intel 64 since can vectorize with libsvml
      - Used for some division and square root implementations

- No official standard (yet) dictates accuracy or how results should be rounded (except for division & sqrt)

-fp-model precise helps generate consistent math calls
  - eg within loops, between kernel & prolog/epilog
  - Remove or reduce dependency on alignment
  - May prevent vectorization unless use –fast-transcendentals
    - When may differ from non-vectorized loop
Select minimum precision

- Currently for libsvml (vector); scalar libimf normally “high”
  - `-fimf-precision= <high|medium|low>`
    - Default is off (compiler chooses)
    - Typically high for scalar code, medium for vector code
    - “low” typically halves the number of mantissa bits
      - Potential performance improvement
    - “high” ~0.55 ulp; “medium” < 4 ulp (typically 2)

`-fimf-arch-consistency= <true | false>`

- Will produce consistent results on all microarchitectures or processors within the same architecture
- Run-time performance may decrease
- Default is false (even with `-fp-model precise` !)
Math Libraries – potential issues

- Differences could potentially arise between:
  - Different compiler releases, due to algorithm improvements
    - Use -fimf-precision
    - another workaround, use later RTL with both compilers
  - Different platforms, due to different algorithms or different code paths at runtime
    - Libraries detect run-time processor internally
    - Independent of compiler switches
    - use -fimf-arch-consistency=true

- Expected accuracy is maintained
  - 0.55 ulp for libimf
  - < 4 ulp for libsvml (default for vectorized loops)

- Adherence to an eventual standard for math functions would improve consistency but at a cost in performance.
Intel® Math Kernel Library

• Linear algebra, FFTs, sparse solvers, statistical, ...
  – Highly optimized, vectorized
  – Threaded internally using OpenMP*
  – By default, repeated runs may not give identical results

• Conditional **BitWise Reproducibility** (new)
  – Repeated runs give identical results under certain conditions:
    – Same number of threads
    – OMP_SCHEDULE=static (the default)
    – Same OS and architecture (e.g. Intel 64)
    – Same microarchitecture, or specify a minimum microarchitecture
    – Consistent data alignment
  – Call `mkl_cbwr_set(MKL_CBWR_COMPATIBLE)`
  – Or set environment variable `MKL_CBWR_BRANCH="COMPATIBLE"
  – In Intel® Composer XE 2013
Intel® Threading Building Blocks

• A C++ template library for parallelism
  – Dynamic scheduling of user-defined tasks
  – Supports parallel_reduce() pattern
  – Repeated runs may not give identical results

• “Community preview” feature for reproducibility:
  – parallel_deterministic_reduce()
  – In Intel® Composer XE 2013
  – Repeated runs give identical results provided the user-supplied body yields consistent results
    – Independent of the number of threads
      – Simple partitioner always breaks up work in the same way
    – But results may differ from a serial reduction
  – May be some impact on performance
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Floating-Point Behavior on the Intel® Xeon Phi™ Coprocessor

• Floating-point exception flags are set by KCi vector instructions
  – the flags can be read
  – unmasking and trapping is not supported.
  – attempts to unmask will result in seg fault
  – -fpe0 (Fortran) and -fp-trap (C) are disabled
  – -fp-model except or strict will yield (slow!) x87 code that supports unmasking and trapping of floating-point exceptions

• Denormals are supported by KCi (but slow, like host)
  – Needs -no-ftz or -fp-model precise (like host)

• 512 bit vector transcendental math functions available (SVML)
  – Division and square root implementations still settling down
  – Both SVML and fast inlined divide and sqrt sequences available
  – Many options to select different implementations
  – See Differences in floating-point arithmetic between Intel(R) Xeon processors and the Intel Xeon Phi(TM) coprocessor for details and status
Comparing Floating-Point Results between Intel® Xeon processors and the Intel® Xeon Phi™ Coprocessor

• Different architectures – expect some differences
  – Different optimizations
  – Use of fused multiply-add (FMA)
  – Different implementations of math functions

• To minimize differences (e.g. for debugging)
  – Build with –fp-model precise  (both architectures)
  – Build with –no-fma                 (Intel® MIC architecture)
  – Select high accuracy math functions
    – (e.g. -fimf-precision=high; default with –fp-model precise )
  – Choose reproducible parallel reductions (slides 15 & 28)
    – Or run sequentially, if you have the patience...
  – Remember, the true uncertainty of your result is probably much greater!
Further Information

- Microsoft Visual C++* Floating-Point Optimization
- The Intel® C++ and Fortran Compiler Documentation, “Floating Point Operations”
- “Consistency of Floating-Point Results using the Intel® Compiler”
- “Differences in Floating-Point Arithmetic between Intel® Xeon® Processors and the Intel® Xeon Phi™ Coprocessor”
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Notice revision #20110804
## Quick Overview of Primary Switches

<table>
<thead>
<tr>
<th>Primary Switches</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>/fp:keyword</td>
<td><strong>fast</strong>[=1</td>
</tr>
<tr>
<td>-fp-model <strong>keyword</strong></td>
<td>Controls floating point semantics</td>
</tr>
<tr>
<td>/Qftz[-]</td>
<td>Flushes denormal results to Zero</td>
</tr>
</tbody>
</table>

**Some Other switches**

| /Qfp-speculation **keyword**     | **fast**, safe, strict, off floating point speculation control              |
| -fp-speculation **keyword**      |                                                                            |
| /Qprec-div[-]                    | Improves precision of floating point divides                               |
| /Qprec-sqrt[-]                   | Improves precision of square root calculations                             |
| /Qfma[-]                         | Enable[Disable] use of fma instructions                                     |
| /Qfp-trap:...                    | Unmask floating point exceptions (C/C++ only)                              |
| -fp-trap=common                  |                                                                            |
| /fpe:0                           | Unmask floating point exceptions (Fortran only)                            |
| /Qfp-port                        | Round floating point results to user precision                             |
| /Qprec                            | More consistent comparisons & transcendentals                               |
| /Op[-]                           | Deprecated; use /fp:source etc instead                                      |
| -mp [-nofltconsistency]          |                                                                            |
## Floating-point representations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Single</th>
<th>Double</th>
<th>Quad or Extended Precision (IEEE_X)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Format width in bits</td>
<td>32</td>
<td>64</td>
<td>128</td>
</tr>
<tr>
<td>Sign width in bits</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>mantissa</td>
<td>23 (24 implied)</td>
<td>52 (53 implied)</td>
<td>112 (113 implied)</td>
</tr>
<tr>
<td>Exponent width in bits</td>
<td>8</td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td>Max binary exponent</td>
<td>+127</td>
<td>+1023</td>
<td>+16383</td>
</tr>
<tr>
<td>Min binary exponent</td>
<td>-126</td>
<td>-1022</td>
<td>-16382</td>
</tr>
<tr>
<td>Exponent bias</td>
<td>+127</td>
<td>+1023</td>
<td>+16383</td>
</tr>
<tr>
<td>Max value</td>
<td>$\approx 3.4 \times 10^{38}$</td>
<td>$\approx 1.8 \times 10^{-308}$</td>
<td>$\approx 1.2 \times 10^{-4932}$</td>
</tr>
<tr>
<td>Value (Min normalized)</td>
<td>$\approx 1.2 \times 10^{-38}$</td>
<td>$\approx 2.2 \times 10^{-308}$</td>
<td>$\approx 3.4 \times 10^{-4932}$</td>
</tr>
<tr>
<td>Value (Min denormalized)</td>
<td>$\approx 1.4 \times 10^{-45}$</td>
<td>$\approx 4.9 \times 10^{-324}$</td>
<td>$\approx 6.5 \times 10^{-4966}$</td>
</tr>
</tbody>
</table>
### Special FP number representations

**Single precision representations**

<table>
<thead>
<tr>
<th></th>
<th>1 Sign bit</th>
<th>8 Exponent bits</th>
<th>(1)+23 Significand bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>zero</td>
<td>0 or 1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>denormalized</td>
<td>0 or 1</td>
<td>0</td>
<td>(0.)xxxxx...</td>
</tr>
<tr>
<td>normalized</td>
<td>0 or 1</td>
<td>1-254</td>
<td>(1.)xxxxx...</td>
</tr>
<tr>
<td>infinity</td>
<td>0 or 1</td>
<td>255</td>
<td>0</td>
</tr>
<tr>
<td>Signalling NaN (SNaN)</td>
<td>No meaning</td>
<td>255</td>
<td>(1.)0xxxx...</td>
</tr>
<tr>
<td>Quiet Nan (QNaN)</td>
<td>No Meaning</td>
<td>255</td>
<td>(1.)1xxxx...</td>
</tr>
</tbody>
</table>
Flush-To-Zero and Denormal FP Values

- A **normalized** FP number has leading binary bit and an exponent in the range accommodated by number of bits in the exponent.

- example:

  \[ 0.171865_{10} = 1/8 + 1/32 + 1/64 \]

  \[ = 0.00101\_2 \]

  normalized = \( 1.0\_2 \times 2^{-3} \)

- Exponent is stored in 8 bits single or 11 bits double: mantissa in 23 bits single, 52 bits double

- exponent biased by 127 (single precision)

- leading sign bit – normalized “1.” bit implied, not physically stored ( 1.011 stored as 011 )

  \[ 0\ 01111100\ 011000000000000000000000 \]
Flush-To-Zero and Denormal FP Values

- What happens if the number is close to zero BUT exponent $X$ in the $2^{-X}$ won’t fit in 8 or 11 bits?
- $2^{-128}$ for example in single precision
- Cannot represent in a NORMALIZED fashion:
  - $1/2^{127} = 0.00...001_2$ (126 zeros after the binary point and a binary 1)
  - $= 1.0_2 \times 2^{-128}$
- But -128 won’t fit in a 127 biased 8-bit exponent value!
- Solution: DENORMAL representation
  - Exponent is -126 (all zeros), NO implied leading 1.
  - 0 00000000 10000000000000000000000
Flush-To-Zero and Denormal FP Values

• “Underflow” is when a very small number is created that cannot be represented. “gradual underflow” is when values are created that can be represented as denormal
• Denormals do not include as many significant digits
• Gradual loss of precision as denormal values get closer to zero

• OK, fine, *I like these denormal numbers, they carry some precision* – why are denormals an issue?
  – **UNFORTUNATELY** denormals can cause 100x loss of performance
• Solution: set any denormal to zero: **FLUSH TO ZERO**
  – Keeps performance up, tradeoff is some loss of precision and dynamic range
–prec-div and –prec-sqrt Options

• Both override the –fp-model settings
• Default is –no-prec-sqrt, and somewhere between –prec-div and –no-prec-div

[-no]-prec-div /Qprec-div[-]
• Enables[disables] various divide optimizations
  – $x / y \Leftrightarrow x \times (1.0 / y)$
  – Approximate divide and reciprocal

[-no]-prec-sqrt /Qprec-sqrt[-]
• Enables[disables] approximate sqrt and reciprocal sqrt
-[no-]fast-transcendental

The compiler frequently optimizes calls of math library functions (like exp, sinf) in loops
- Uses SVML (short vector math library) to vectorize loops
- Uses the XMM direct call routines,
  e.g. \texttt{exp} → \texttt{__libm_sse2_exp} (IA-32 only)
  - May sometimes use fast in-lined implementations

This switch “-[no]fast-transcendental” can be used to overwrite default behavior
- Behavior related to settings of fp-model and other switches – see reference manual !!
gcc options

• -ffast-math implies
  – -fno-math-errno
  – -funsafe-math-optimizations
  – -ffinite-math-only
  – -fno-rounding-math
  – -fno-signaling-nans
  – -fcx-limited-range
  – & sets __FAST_MATH__

• -funsafe-math-optimizations implies
  – -fno-signed-zeros
  – -fassociative-math
  – -fno-trapping-math
  – -freciprocal-math
Math Functions on the Intel® Xeon Phi™ Coprocessor

• Faster, more approximate versions of math functions can still be obtained with -fp-model precise by adding
  - fast-transcendentals -no-prec-div -no-prec-sqrt
  - See Differences in floating-point arithmetic between Intel(R) Xeon processors and the Intel Xeon Phi(TM) coprocessor for details and status

• Switches for finer control of math function accuracy:
  – -fimf-precision=<high|medium|low> [:func1,func2,...]
  – -fimf-max-error
  – -fimf-accuracy-bits
  – -fimf-absolute-error
  – -fimf-domain-exclusion
Math Functions on the Intel® Xeon Phi™ Coprocessor

• Math functions have special branches and code to handle “exceptional” arguments
  – Faster versions possible if this can be skipped
• -fimf-domain-exclusion= <value>; the bits of <value> indicate domains for which the compiler need not generate special code
  – 1 extreme values (close to singularities or infinities; denormals)
  – 2 NaNs
  – 4 infinities
  – 8 denormals
  – 16 zeros
  – E.g. -fimf-domain-exclusion=31 excludes all of these for all functions
• Can be restricted to specific functions, e.g.
  – -fimf-domain-exclusion=15:/sqrt,sqrtf gives fast, inlined versions of single & double precision square root
• -fp-model-fast=2 implies -fimf-domain-exclusion=15