Using MMX™ Instructions to Implement the Levinson-Durbin Algorithm

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1.0. INTRODUCTION

The Intel Architecture (IA) media extensions include single-instruction, multi-data (SIMD) instructions. This application note presents examples of code that exploit these instructions. Specifically, the levinsonmmx function presented here illustrates how to use the new MMX technology multiply-and-add instruction (PMADDWD) to perform matrix multiplication more efficiently. The performance improvement relative to traditional IA code is due to the ability to efficiently perform multiple multiply-and-add operations in fewer cycles. To perform a signed Q15 multiply-and-add would use two IA instructions, IMUL and ADD, and would take as many as 13 cycles. Using the MMX technology PMADDWD instruction, you can perform four word multiplies and two doubleword adds in three cycles. Performance gain can also be attributed to the fact that the MMX instructions operate on packed 64-bit values instead of 32-bit values.

2.0. THE LEVINSON-DURBIN ALGORITHM

The amount of data which represents a human voice or sound is usually too large to store on a typical PC. Therefore, encoding the sound and storing only a partial set of the data would be more practical. Voice encoding is one of the applications in which the Levinson-Durbin algorithm is used. This algorithm generates both a set of prediction coefficients and reflection coefficients in a recursive manner as follows:

\[
\begin{align*}
    a_n &= K_n = - \frac{r(m) + r_{n-1} a_{n-1}}{r(0) + r_{n-1} a_{n-1}^b} \\
    a_m(k) &= a_{m-1}(k) + K_m a_{m-1}(m-k) \quad k = 1, 2, \ldots, m-1 \\
    m &= 1, 2, \ldots, p
\end{align*}
\]

where \( a \) is the prediction coefficient array, \( K \) is the reflection coefficient array, and \( r \) is the input vector.

Note: The superscript \( b \) denotes the vector with elements taken in reverse order.

The integer algorithm consists of three sections:

1. The section which performs the matrix multiplication of \( R_n = (r[] \ast a[]) \) and \( R_d = (r^b[] \ast a[]) \) to generate the numerator and denominator values for the division operation \((r^b[] \ast a[])\) to generate the numerator and denominator values for the division operation.

2. The section which performs the division to calculate the prediction and reflection coefficients for the \( m^{th} \) iteration of the loop.

3. The section which recalculates the prediction coefficients for the \((m+1)^{th}\) iteration of the main loop.

2.1. Input and Output Data Representation

The input and output of this version of the Levinson-Durbin algorithm is represented in fixed-point notation. The input matrix \( rf[] \) and the resultant reflection coefficients \( k[] \) are represented as Q15.
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fractions stored in an array of short integers of size \( m \). The prediction coefficients \( a[i] \) are represented as Q13 fractions also stored in an array of short integers of size \( m \). If two (2) signed Q15 fractions are multiplied together, the result will be a Q30 signed fraction. If a Q15 signed fraction is multiplied by a Q13 signed fraction, the result is a Q28 signed fraction. To determine the number of bits assigned to the fractional part of the result of a multiplication, add the number of bits assigned to the fractional parts of each multiplicand. To determine the number of bits assigned to the fractional part of the result of a division, subtract the number of bits assigned to the denominator from the number of bits assigned to the numerator. Refer to Figure 1 for further clarification.

2.2. Error Correction Techniques

There are various areas in this algorithm where errors can occur. These errors are due to noise in the input data and performing integer instead of floating-point arithmetic. This version of the Levinson-Durbin algorithm performs two error correction techniques to minimize the overall amount of error found in the output results. The first technique minimizes the error introduced by using integer arithmetic. This is accomplished by rounding results to the nearest digit of precision prior to converting from one precision to another (i.e. Q28 to Q13). Refer to Example 2 for a graphical representation of fixed point conversion techniques.

**Example 2. Fixed Point Conversion**

Converting from a Q28 to a Q13 signed fraction:
0x0A234238 Q28 signed fraction + 0x00004000 rounding factor 0x0A238238 rounded result >> 15 0x00001447 conversion result

The second technique brings a set of input data back into a usable range for the Levinson-Durbin algorithm. Input data can be determined to be not stable if any of the reflection coefficients produced are
greater than one. To stabilize the input data, a scaling factor was used to scale both the prediction and reflection coefficients. This number may vary across applications and may need to be modified. For the system which was used to verify the MMX code optimized Levinson-Durbin algorithm, the appropriate scaling factor was was 0x7ff8.
3.0. IMPLEMENTING LEVINSON-DURBIN USING MMX™ INSTRUCTIONS

There were numerous areas within the Levinson-Durbin algorithm which could be easily adapted and optimized for use with the MMX instruction set. There was also one area which could not. The remainder of this section discusses each part of the Levinson-Durbin algorithm and the optimizations and MMX technology tricks used during programming.

3.1. The Inner Loop: Calculating Rₙ and Rₐ

The formulas used to calculate Rₙ and Rₐ are as follows:

\[
\begin{align*}
Rₙ &= r(m-i) \times a(i) \quad \text{for } i=0 \text{ to } m-1 \\
Rₐ &= r(i) \times a(i) \quad \text{for } i=0 \text{ to } m-1
\end{align*}
\]

where \(r()\) is the input matrix and \(a()\) is the prediction coefficient matrix.

These formulas are simple vector multiplications, but when using strictly IA integer instructions may take many cycles to calculate. The MMX instruction set supplies a multiply-add instruction (PMADDWD) which performs four word multiplies and two double-word adds in three cycles. This instruction can be pipelined with any MMX instruction, including another PMADDWD instruction. The result of a PMADDWD instruction can only be used by other instructions issued three clocks later. If it is used prior to the three clock latency, a stall will occur until the result is ready.

The inner loop which calculates \(Rₙ\) and \(Rₐ\) uses the PMADDWD instruction to perform four word multiplies and two doubleword adds for \(Rₙ\) and \(Rₐ\) with each iteration. The result of each multiply and add instruction is two doubleword Q28 signed fixed point fractions stored in MM0 (\(Rₐ\)) and MM2 (\(Rₙ\)). Each of these values must be added to an accumulator, which keeps a running summation of the current result (MM6 is used for \(Rₐ\) and MM7 is used for \(Rₙ\)).

In order to use the PMADDWD instruction within the inner loop to perform four word multiplies and two doubleword adds, two assumptions were made. The first is that the prediction coefficient array is initialized to zero. This prevents incorrect results for calculations of \(Rₙ\) and \(Rₐ\) which include three or fewer input values. The second is that the size (in bytes) of both the input vector and the prediction coefficient vector must be divisible by eight. This is to prevent reading past the end of either vector since four input values are read for each iteration of the loop.

3.2. Inner Loop Analysis

Example 4 contains the code for the inner loop of the Levinson-Durbin algorithm.

\[
\begin{align*}
\text{Example 4. Inner Loop Code Segment For Calculating Four Elements per Iteration} \\
calcRnRd: \\
1. & \text{movq mm2, [edx][eax*2]} & \text{get the next 4 rT() matrix entries} \\
2. & \text{pmaddwd mm0, mm1} & \text{calculate } (r(i) \times a(i)) + (r(i+1) \times a(i+1)) \\
& \text{add eax, eax;} \quad \text{increment the aMatrix address} \\
3. & \text{pmaddwd mm2, mm1} & \text{calculate } (r(i) \times a(i)) + (r(i+1) \times a(i+1)) + \\
& \text{add eax, eax;} \quad \text{increment the aMatrix address} \\
4. & \text{pmaddwd mm2, mm1} & \text{calculate } (r(i) \times a(i)) + (r(i+1) \times a(i+1)) + \\
\end{align*}
\]
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5. Two penalties occur during this clock cycle. The first is a pmaddwd stall since the instruction on line 8 uses the result of the pmaddwd instruction issued on line 2. The second penalty occurs because eax is incremented on line 3 and used the next cycle.

6. movq mm1, [ecx][eax*2] ; get the next 4 a() matrix entries
7. paddd mm6, mm0 ; accumulate Rd
8. movq mm0, [esi][eax*2] ; get the next 4 r() matrix entries
9. paddd mm7, mm2 ; accumulate Rn
10. cmp eax, ebx ; compare i and m
11. jl calcRnRd ; if (i < m) then keep incrementally calculating Rn and Rd

Assuming all data is present in the data cache, the inner loop which calculates \( R_n \) and \( R_d \) varies in the number of cycles it takes to execute. If the data being read from the reverse input matrix is aligned, no penalty is incurred and the loop executes in six cycles per pass. If the data being read is unaligned, the read incurs a three cycle penalty, increasing the overall loop time from six to nine cycles per pass. Data alignment is extremely critical to the number of cycles in which the loop executes. If the input matrix and the prediction coefficient matrix are not aligned on quad word boundaries, accesses to these matrices will also incur an unaligned penalty of three clock cycles.

Within the inner loop is a one clock cycle penalty which is caused by two violations of the pairing rules. The first penalty occurs on line 7 where a quad word move is issued. This instruction uses the EAX register to generate an index into the a matrix. Exactly one cycle prior to issuing the MOVQ instruction, EAX was used as a destination register on line 3. Pairing rules state that any integer register can not be used to address memory for two clocks after it has been modified. This is referred to as an address generation interlock (AGI). Therefore a penalty of one clock cycle is incurred. The second penalty occurs on line 8 where a packed add is issued. This instruction uses MM0 as an operand. MM0 contains the result of the PMADDWD instruction issued on line 2. Pairing rules state that the result of a multiply instruction can only be used by other instructions issued three clocks later. Therefore, a penalty of one clock cycle is incurred. Since both penalties occur in the same cycle, only a one clock cycle stall occurs instead of two.

This loop was optimized to support efficient calculations of both the prediction and reflection coefficients to an order of 10. For orders of 16 or greater, this loop can be optimized even further by unrolling the loop one more time. This would be done by performing eight word multiplies and four doubleword adds for both \( R_n \) and \( R_d \) each time through the loop. It would eliminate half of the loop overhead and may also have the potential of hiding the multiply-add and AGI pipe stall at instruction 8. See Example 5 for an example.

**Example 5. Inner Loop Code Segment For Calculating Eight Elements per Iteration**

```
calcRnRd:
1. movq mm2, [edx][eax*2] ; get the next 4 rT() matrix entries
2. pmaddwd mm0, mm1 ; calculate \( (r(i) \times a(i)) + (r(i+1) \times a(i+1)) + (r(i+2) \times a(i+2)) + (r(i+3) \times a(i+3)) \)
```
3.3. Obtaining the Reverse of the Input Vector

The formula to compute \( R_n \) uses the values of the input vector in the reverse order. Three options were considered when deciding how to obtain this vector:

1. Create the reverse vector prior to the main loop.
2. In the inner loop to calculate \( R_n \) and \( R_d \), read the input vector and rearrange the data for the calculation.
3. Add an input vector element to the front of a local array with each iteration of the main loop. Use this new array to calculate \( R_n \). This option is available since for each iteration of the main loop, only one new input vector element is used to calculate \( R_n \).

Option 3 was chosen since this read/store operation could be paired with other non-paired instructions prior to executing the inner loop. Therefore, the reverse of the input matrix was obtained "free." Both options 1 and 2 were investigated and discarded because of the extra cycles added to the overall performance of the code. Option 1 added extra instructions and loop overhead to the function. Option 2 added extra cycles within the inner loop to rearrange data appropriately for the calculation of \( R_n \).

3.4. Getting Ready for the Integer Divide

After the execution of the inner loop, \( R_n \) and \( R_d \) have their results accumulated in MM7 and MM6, respectively. These registers each have two doubleword Q28 signed fractions which need to be separated and added together to obtain the final result. A copy of each accumulator is made and then the packed-shift-right-logical-quad (PSRLQ) instruction is used to shift the high doubleword into the low doubleword.
position. This is added to the accumulator to obtain the final $R_n$ and $R_d$ result in the low doubleword position. Refer to Example 6 for a code example. In this example, there are also statements which get the data ready for the division in the next code segment. This includes negating $R_n$ and clearing registers MM3 and MM0.

**Example 6. Finishing the $R_n$ and $R_d$ Calculation**

```assembly
Example 6. Finishing the $R_n$ and $R_d$ Calculation

calncw A:
1. movq mm0, mm6  ; make a copy of the partial Rd
2. movq mm1, mm7  ; make a copy of the partial Rn
3. movd mm4, round_factor ; load the rounding factor for later
4. psrlq mm1, 32   ; put the high doubleword of Rn into
                    ; the low doubleword position
5. paddd mm7, mm1  ; do the final Rn calculation
6. pxor mm3, mm3  ; zero out mm3 to negate Rn for the
                    ; divide
7. psubd mm3, mm7  ; negate Rn prior to the divide
8. psrlq mm0, 32   ; put the high doubleword of Rd into
                    ; the low doubleword position
9. paddd mm6, mm0  ; do the final Rd calculation
10. pxor mm1, mm1  ; zero out mm1 for unpacking the
                   ; rounding factor later
```

3.5. Calculating the $m^{th}$ Order Reflection and Prediction Coefficients

The $m^{th}$ order reflection and prediction coefficients are calculated by dividing $R_n$ by $R_d$. This integer division can not be avoided since the denominator changes for each iteration of the main loop. It takes 43 clock cycles every time that it is executed. If the denominator value remained constant with each iteration of the outer loop, one divide could have been executed prior to entering the main loop to determine $1/R_d$. This value could then be used in an integer multiplication using the IMUL instruction and would take 10 cycles to execute. The IMUL instruction would have to be used in this instance since the MMX instruction set does not supply a doubleword multiply instruction. A floating point divide could also be executed in place of the fixed point integer divide, but would incur a 100 clock cycle penalty of switching from floating point code to MMX code. Therefore it was not used.

3.6. Recalculating the Prediction Coefficients

This segment of code recalculates the prediction coefficients from 0 through $m-1$ based on the newly calculated $m^{th}$ coefficient. The pseudocode is as follows:

**Example 7. Algorithm For Recalculating the Prediction Coefficients**

```assembly
for ( i = 1; i < m; i++)
    b[i] = a[i] + (K[m] * a[m-i]);
for ( i = 1; i < m; i++)
    a[i] = b[i];
```

To avoid needing the temporary array b, the MMX code calculates both the new $a[i]$ and $a[m-i]$ with each iteration of the loop. The MMX instruction set does not implicitly provide a double-word multiply. Therefore, the PMADDWD instruction was used. Refer to Figure 2 for a graphical representation of the data setup. After the data has been setup, the instruction PMADDWD MM0, MM2 is issued to calculate $K[m]*a[i]$ and $K[m]*a[m-i]$. The result of this multiply-add is a Q28 signed fraction. It is then added to MM3, which contains $a[i]$ in the low doubleword and $a[m-i]$ in the high doubleword, each represented as Q28 signed fractions. This addition obtains the new $a[i]$ and $a[m-i]$ values.
An approximate overall performance of this version of the Levinson-Durbin algorithm can be calculated based on the performance of three sections of the code: the calculation of $R_n$ and $R_d$, the calculation of $K[m]$, and the re-calculation of the prediction coefficients. The following discussion about performance assumes all data is present in the data cache and that all data is properly aligned.

There are two cycle count numbers which are important to take into consideration when approximating the number of cycles it takes to calculate $R_n$ and $R_d$. When accesses to the reverse of the input vector are aligned, it takes six clock cycles per iteration to multiply and accumulate four values for both $R_n$ and $R_d$. When accesses to the reverse of the input vector are unaligned, it takes nine clock cycles per iteration. Since half of all accesses to the reverse of the input vector are aligned and half are unaligned, on average it takes 7.5 cycles to calculate four values for both $R_n$ and $R_d$ or .9375 clock cycles for one value.

To calculate $a/m]$ and $K[m]$ and to set up register values prior to re-calculating the prediction coefficients takes approximately 64 clock cycles. The majority of these cycles can be directly attributed to the integer divide which executes in 43 clock cycles. As mentioned previously, this integer divide can not be avoided.

For the re-calculation of the prediction coefficients, there are two cycle count numbers which must be taken into consideration. When accesses to the prediction coefficient array are aligned, it takes 14 clock cycles to recalculate two elements. When accesses to the prediction coefficient array are unaligned, it takes 20 clock cycles to recalculate two elements. Since half of all accesses to the prediction coefficient array are aligned and half are unaligned, on average it takes 17 cycles to recalculate two elements or 8.5 clock cycles to recalculate one.

Applying these cycle count numbers to an application which calculates $pSize$ reflection coefficients using an input vector of size $pSize+1$, an approximate overall performance can be obtained as follows:

For the C language version of the Levinson-Durbin algorithm, refer to Appendix A. For the complete MMX code optimized version of the Levinson-Durbin algorithm, refer to Appendix B.

The MMX code optimized version is reentrant but the user may wish to dispose of the local variables and pass them in as arguments to the procedure if stack usage is an issue.
APPENDIX A: "C' Version of the Levinson-Durbin Algorithm

/ *******************************************************************************
* Description:
* Levinson-Durbin is the scalar (not MMX code) version of the Levinson-Durbin
* algorithm. It is used to calculate the reflection and prediction coefficients
* of
* a given set of normal equations.
* *
* Inputs:
* r short int *  a pointer to the first element
*     of the input 'r' matrix
* a short int *  a pointer to the output
* prediction coefficients array
* k short int *  a pointer to the output
* reflection coefficients array
* p short int  the number of reflection coefficients
* to solve for (typically 10 or 16).
* *
*********************************************************************************/
void Levinson_Durbin (short *r, short a*, short *k, short p)
{   // begin Levinson_Durbin()
    short i,   // inner loop index
    m,   // outer loop index
    b[11];   // temporary vector to store the prediction coefficients
    long Rn,   // inner loop numerator accumulator
    Rd,   // inner loop denominator accumulator
    temp;   // temporary variable used for intermediate calculations
    // initialize a[0] to 1/4
    a[0] = 8192;
    // For each order, calculate a new prediction and reflection coefficient
    for (m = 1; m < p + 1; m++)
    {
        // Initialize the numerator and denominator accumulators to zero
        Rn = Rd = 0;
        // Calculate the numerator and denominator values for the integer
        division
        for (i = 0; i < m; i++)
        {
            Rn = Rn + (r[m-1] * (long)a[i]);
            Rd = Rd + (r[i] * (long)a[i]);
        }
        // Calculate the reflection coefficient k[m]. Round the Q28 number
        prior to converting it to a Q15 number. Also, scale it by the scaling
        factor
        k[m] = -Rn / ((Rd + 0x4000) >> 15);
        k[m] = (((long)k[m] * 0x7ff8) + 0x4000) >> 15;
        // Calculate the new prediction coefficient by converting k[m] from
        // a Q15 to a Q13 number
        b[m] = (k[m] + 0x2) >> 2;
        // Calculate the new prediction coefficients for the next iteration
        for (i = 1; i < m; i++)
        {   //..
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```
b[m] = (((long)a[i] << 15) + (k[m] * (long)a[m-i]) + 0x4000) >> 15);
// Copy the prediction coefficients from the temporary b[] array to
a[]
     for ( i = 1; i < m+1; i++)
        a[i] = b[i];
}   // end Levinson_Durbin()
```
APPENDIX B: The MMX™ Technology Version of the Levinson-Durbin Algorithm

;**************************************************************************
;* Description:  
;* The purpose of this file is to provide the MMX code for the 
;* levinson-durbin algorithm as an instructional example to those who 
;* are just beginning to code using MMX instructions.
;*
;* Assumptions: 
;* 1. The set of normal equations given are stable 
;* 2. The output matrix 'a' containing the prediction coefficients 
;* is initialized to all 0's.
;* 3. The normal equations are represented by one matrix 'r' which 
;* contains the coefficients gamma(0) through gamma(p) given as 
;* Q15 signed short integers.
;* 4. The resultant reflection coefficients will be returned as 
;* Q15 signed short integers.
;* 5. The resultant prediction coefficients will be returned as 
;* Q13 signed short integers.
;* 6. The rTmatrix defined below is of size (p*2) + 16 bytes.
;**************************************************************************

_TEXT SEGMENT

Declare levinson_mmx as a public routine to allow the 'C' code to 
call it.

PUBLIC levinson_mmx

_levinson_mmx is the MMX optimized code version of the Levinson-Durbin 
algorithm. It is used to calculate the reflection coefficients of 
a given set of normal equations.

* Inputs: 
* rPtr short int * a pointer to the first element 
* of the input 'r' matrix 
* aPtr short int * a pointer to the output 
* prediction coefficients array 
* kPtr short int * a pointer to the output 
* reflection coefficients array 
* pSize short int the number of reflection coefficients 
to solve for (typically 10 or 16).

**************************************************************************

levinson_mmx PROC C USES ebx ecx edx esi, 
rMatrix:PTR WORD, 
aMatrix:PTR WORD, 
kMatrix:PTR WORD, 
pSize:DWORD 

; Declare local variables 
1. rTmatrix - used to store the reverse of rMatrix
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; 2. rTindex - used to store the rTmatrix index
; 3. mSave - used to store the loop counter 'm'
; 4. round_factor - contains the rounding factor 0x4000
; 5. scale_factor - contains the scaling factor 0x7ff8

LOCAL rTmatrix[72]:WORD
LOCAL rTindex:DWORD
LOCAL mSave:DWORD
LOCAL round_factor:DWORD
LOCAL scale_factor:DWORD

; This section of code initializes the local variables round_factor and scale_factor
; and sets up some of the items which need to be done prior to executing the
algorithm
;
mov    round_factor, 4000H ; initialize the rounding
factor
mov    eax, pSize ; get the value of 'p'
mov    scale_factor, 7ff8H ; initialize the scaling factor
mov    esi, aMatrix ; get the pointer to the 'a'
matrix
mov    WORD PTR [esi], 2000H ; store the scaling element
into a[0]
lea    edx, rTmatrix[eax*2] ; get the first rTmatrix index
address
mov    rTindex, edx ; save the first rTmatrix
address
mov    ebx, 1 ; initialize the main loop
counter 'm'
;
; This section of code sets up the pointer to
; the input array 'r' and rTmatrix used to calculate Rn and Rd. It also
; performs the test:
; while (p >= m)
;
_levinson_outer_loop:
    mov    esi, rMatrix ; initialize the rMatrix
    cmp    eax, ebx ; is (p >= m) ?
    mov    ecx, [esi][ebx*2] ; get the next rMatrix element
    jl     _levinson_done ; if (p < m) we are done!
;
; This section of code sets up registers used to calculate Rn and Rd. It also
; incrementally builds the rTmatrix with each iteration of the main levinson loop,
; adding one more rMatrix element to the front of the array. The reverse of the
; input matrix 'r' is needed when calculating Rn. Rn uses the elements 1 to m of
; rMatrix in reverse as one of the multiplicands with aMatrix elements as the other.
;
_setup_RnRd_calc:
    mov    eax, 0 ; initialize the loop counter
to 0
    mov    [edx], cx ; store the next rTmatrix
element
    mov    ecx, aMatrix ; get the pointer to the 'a'
matrix
    movq   mm0, [esi] ; get the first quad word of
    'r'
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pxor mm6, mm6 ; initialize Rd to 0
movq mm1, [ecx] ; get the first quad word of aMatrix

pxor mm7, mm7 ; initialize Rn to 0
cmp eax, ebx ; is (i < m) ?
jge _calc_new_A ; if not, calculate Rn and Rd

; This section of code calculates Rn and Rd. The calculation used is:
; while (i < m) {
;   Rn = Rn + (rTmatrix[i]*aMatrix[i]);
;   Rd = Rd + (rMatrix[i+1]*aMatrix[i]);
; }
; where Rn and Rd are long, Q28 signed fractions, rTmatrix,
; and rMatrix are short, Q15 signed fractions and aMatrix contains
; short, Q13 signed fractions. Up to 4 multiply/adds are done at a time
; for both Rn and Rd.

_calc_RnRd:
movq mm2, [edx][eax*2] ; get the next quad word of rTmatrix
pmaddwd mm0, mm1 ; calculate 'r' * aMatrix quad

add eax, 4 ; increment the loop counter
pmaddwd mm2, mm1 ; calculate rTmatrix * aMatrix quad

mul/add
movq mm1, [ecx][eax*2] ; get the next aMatrix quad word
paddwd mm6, mm0 ; Rd = Rd + (rMatrix * aMatrix) quad

word
movq mm0, [esi][eax*2] ; get the next rMatrix quad word
paddwd mm7, mm2 ; Rn = Rn + (rTmatrix * aMatrix) quad

word
cmp eax, ebx ; is (i < m)?
jl _calc_RnRd ; if so, then keep incrementally calculating

; Upon leaving the loop that calculates Rn and Rd, Rd (mm6) and Rn (mm7) contain two
; signed Q28 fractions which need to be added together to get the final Rn
; and Rd result. This code performs this calculation.

_calc_new_A:
movq mm0, mm6 ; make a copy of the partial Rd result
movq mm1, mm7 ; make a copy of the partial Rn result
movd mm4, round_factor ; load the rounding factor
psrlq mm1, 32 ; shift the Rn copy over by 32 for the add
paddwd mm7, mm1 ; do the last Rn calculation
pxor mm3, mm3 ; clear this MMX register
psubd mm3, mm7 ; negate Rn prior to doing the divide
psrlq mm0, 32 ; shift the Rd copy over by 32 for the add

; This section of code performs the calculation of a[m] (the prediction coefficient)
; and k[m]
; (the reflection coefficient). The calculation is as follows:
;   K[m] = Rn / ((Rd + 0x4000) >> 15);
; This step produces a Q15 signed result
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; To obtain a short, Q15 signed reflection coefficient we must divide the
; Q28 signed numerator by a Q13 signed denominator. This
; will provide the desired result with the correct precision. The integer division
; can not be avoided since for every 'm' the denominator will be different.
;
; padd dm6, dm0
; pxor dm1, dm1
; movd eax, dm3
; move the numerator to an integer

; shifting
paddm dm6, dm4
movd mm3, scale_factor
; setup mm3 with a scaling factor

; 0x7ff8
psrad mm6, 15
; perform the right shift of the
denominator

; mov esi, aMatrix
mov edx, eax
; make edx the sign extension of eax
sar edx, 31
; complete the sign extension for the
idiv
mov mSave, ebx
; save the value of the main loop
counter 'm'
mov ecx, mm6
; move the denominator to an integer
mov ecx, ecx
register

; punpcklwd mm4, mm1
; prepare mm4 to be used as the
; rounding
prediction
mov ecx
; perform the 32-bit integer division

; This section of code scales the new reflection/prediction coefficient, obtains the
correct
; precision for the prediction coefficient (Q13), and stores the results into the 'a'
; matrix and 'K' matrix. The calculations it performs are as follows:
; K[m] = ((K[m] * 0x7ff8) + 0x4000) >> 15;
; This step scales the reflection/prediction coefficient by .999x
; a[m] = (K[m] + 0x2) >> 2;
; This step produces a Q13 signed prediction coefficient.
; It also performs some setup for re-calculating the prediction coefficients.

mov edx, kMatrix
mov ecx, eax
; coefficients
mov ecx, ecx
; perform the idiv result to an MMX

pxor mm5, mm5
; clear out mm5
sub ebx, 1
; dec 'm' since the loop bounds are 0
and m-1

pmaddwd mm2, mm3
; scale K[m] by 0x7ff8
movd mm0, 2[esi]
; get a[1] for loop
punpckldq mm4, mm4
; complete the rounding register setup
movd mm1, [esi][ebx*2]
; get a[m-1] for loop preparation
movq mm3, mm0
; make a copy of a[1] through a[4]
mov edx, kMatrix
; get the kMatrix pointer
paddd mm2, mm4
; round K[m] prior to shifting it
mov ecx, 1
; initialize the loop counter
psrad mm2, 15
; shift K[m] to obtain a Q15 fraction
mov eax, mm2
; move K[m] to an integer register
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```
psslq mm0, 32 ; move a[1] and a[2] to the upper doubleword

mov 2[edx][ebx*2], ax ; store the idiv result into K[m]
add eax, 2 ; round a[m] prior to shifting
sar eax, 2 ; shift a[m] to obtain a Q13 fraction
punpcklwd mm2, mm5 ; set an MMX register to 0, a[m], 0, a[m] as
                ; the a[m] multiplicand used below
mov 2[esi][ebx*2], ax ; store the idiv result into a[m]
punpckldq mm2, mm2 ; finish setting up the 0, a[m], 0, a[m]
                ; register
                ; is (i >= m) ?
jge _done_alter_A ; if so, then jump past this loop

; This section of code calculates the new prediction coefficients to be used for the calculation of the (m+i)’th order reflection coefficient. The following algorithm is used:
; for (i = 0, j = m-1; i < j; i++, j--)
; }
; tempA = K[m] * a[j];
; a[j] = (((long)a[j] << 15) + (K[m] * a[i]) + 0x4000) >> 15;
; a[i] = (((long)a[i] << 15) + tempA + 0x4000) >> 15;
; }
; If, upon exiting this loop, i == j, then we have one more element to do. This code can be found after the end of this loop. Two calculations are done for each loop iteration:
; a[i], a[j]

.Alter A:
por mm0, mm1 ; set mm0 to X, a[i], X, a[j]
punpckldq mm3, mm1 ; set mm3 to X, a[j], X, a[i]
pmaddwd mm0, mm2 ; multiply a[i] and a[j] by K[m]
psllq mm0, 32 ; move a[j] to the low doubleword
            ; convert from Q13 to Q28 signed fraction
sub ebx, 1 ; decrement j
psrad mm3, 1 ; finish the conversion
movd mm5, 2[esi][ecx*2] ; get the next a[i] element
paddd mm3, mm4 ; add the rounding factor to a[i] and a[j]
movd mm1, [esi][ebx*2] ; get the next a[j] element
padd mm0, mm3 ; add a[i] to a[j]*K[m] and a[j] to a[i]*K[m]
movq mm3, mm5 ; make a copy of the next a[i] element
psrad mm0, 15 ; convert the new a[i] and a[j] to Q13 signed fraction
movd edx, mm0 ; move a[i] to an integer register
psrlq mm0, 32 ; move a[j] to the low doubleword
mov [esi][ecx*2], dx ; store a[i]
psslq mm5, 32 ; move the next a[i] to the high doubleword
movd edx, mm0 ; move a[j] to an integer register
movq mm0, mm5 ; move the next a[i] to mm0
mov 2[esi][ebx*2], dx ; store a[j]
inc ecx ; increment i
```
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```
; cmp ecx, ebx ; is (i < j)?
; jl _alter_A ; if so, then keep looping!
;
; If, upon exiting the above loop, i == j, then perform the last calculation on a[i]
; by doing:
; a[i] = (((long)a[i] << 15) + (K[m]*a[i]) + 0x4000) >> 15;
;
_done_alter_A:  
    pmaddwd mm1, mm2 ; perform a[i]*K[m]
    pslld mm3, 16 ; convert from Q13 to Q28 signed fraction
    mov eax, rTindex ; get the rTmatrix current address
    psrad mm3, 1 ; complete the conversion
    mov ebx, mSave ; get the value of 'm', the main loop counter
    paddd mm1, mm4 ; perform a[i] + a[i]*K[m]
    padd mm1, mm3 ; add the rounding factor before
    shifiting
    psrad mm1, 15 ; convert the new a[i] to Q13 signed fraction
    jne _main_loop_done ; if (i != j) then skip this part
    mov edx, mm1 ; move the result to an integer
    mov [esi][ecx*2], dx ; store a[i]
;
; We have now completed finding the (m+1)'th order prediction coefficients and the
; mth reflection coefficient. This section of code sets up registers to check if we need to
; continue with the next 'm'.
;
_main_loop_done:  
    add ebx, 1 ; increment 'm', the main loop counter
    sub eax, 2 ; since we need to add an rMatrix element on
    subtract ; 2 from the address
    mov rTindex, eax ; store this new address back to memory
    mov edx, eax ; setup the rTmatrix array pointer
    mov eax, pSize ; get the value of 'p'
    jmp _levinson_outer_loop ; go to the top of the main loop

_levinson_done:  
    ret 0 ; we are done!

levinson_mmx ENDP
TEXT ENDS
END
```